Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
automatica
Automatica ▮▮▮ (▮▮▮▮) ▮▮▮–▮▮▮
www.elsevier.com/locate/automatica

Brief paper

Robust PID controller tuning based on the constrained particle swarm optimization ☆

Tae-Hyoung Kim*, Ichiro Maruta, Toshiharu Sugie

Department of Systems Science, Graduate School of Informatics, Kyoto University, Uji, Kyoto 611-0011, Japan

Received 12 March 2007; received in revised form 21 August 2007; accepted 30 August 2007

Abstract

This paper proposes a novel tuning strategy for robust proportional-integral-derivative (PID) controllers based on the augmented Lagrangian particle swarm optimization (ALPSO). First, the problem of PID controller tuning satisfying multiple H_∞ performance criteria is considered, which is known to suffer from computational intractability and conservatism when any existing method is adopted. In order to give some remedy to such a design problem without using any complicated manipulations, the ALPSO based robust gain tuning scheme for PID controllers is introduced. It does not need any conservative assumption unlike the conventional methods, and often enables us to find the desired PID gains just by solving the constrained optimization problem in a straightforward way. However, it is difficult to guarantee its effectiveness in a theoretical way, because PSO is essentially a stochastic approach. Therefore, it is evaluated by several simulation examples, which demonstrate that the proposed approach works well to obtain PID controller parameters satisfying the multiple H_∞ performance criteria.

Keywords: PID controller design; H_∞ control; Robustness; Non-convex optimization; Particle swarm optimization

1. Introduction

Tuning strategies for robust proportional-integral-derivative (PID) controllers satisfying the given H_∞ specifications based on optimization approaches have recently received considerable attention. However, the design problem for optimal robust PID controllers based on H_∞ techniques results in a non-convex optimization problem which suffers from computational intractability and conservatism. For such problems, Åström, Panagopoulos, and Hägglund (1998) introduced an iterative procedure using the well-known Newton–Raphson search algorithm to find the PI controller gains within the non-convex domain. However, the choice of good initial conditions is a crucial factor, since it has a considerable effect on a computational efficiency of Newton–Raphson iterations. In Hwang and Hsiao (2002), a derivative gain should be chosen in advance by the designer. Then, based on the given derivative gain, the analytical expressions for describing the boundary of an equality constraint set on proportional and integral gains are derived. The maximum allowable proportional and integral gains are obtained by tracing the boundaries of equality constraint sets using a path-following algorithm. Ho (2003) presented a synthesis of H_∞ PID controllers based on the generalized Hermite–Biehler theorem for complex polynomials which is used to develop a linear programming based optimization algorithm for determining an admissible PID controller. However, a suitable proportional gain should be chosen in advance by the designer to determine the integral and derivative gains. Therefore, the reasonable selections of derivative gain in Hwang and Hsiao (2002) and proportional gain in Ho (2003) are important issues to improve the performance of the developed PID controllers.

From the above observations, it is required to develop a novel tuning strategy of robust PID controller, which can determine

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Masayuki Fujita under the direction of Editor Ian Petersen.

* Corresponding author. Tel.: +81 774 38 3952; fax: +81 774 38 3945.
E-mail addresses: hyoung@robot.kuass.kyoto-u.ac.jp (T.-H. Kim), maruta@robot.kuass.kyoto-u.ac.jp (I. Maruta), sugie@i.kyoto-u.ac.jp (T. Sugie).

0005-1098/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2007.08.017

all controller gains simultaneously by solving an optimization problem subject to multiple constraints on H_∞ specifications. Further, most of the conventional PID controller design techniques are based on simple characterization of system dynamics as first-order or second-order models, and there are very few generally accepted design methods for systems with higher order (Ho & Lin, 2003). Thus, it is one of the important research issues to develop a robust PID controller applicable to higher order systems.

On the other hand, Eberhart and Kennedy (1995) recently proposed a particle swarm optimization (PSO) algorithm which is a swarm intelligence technique and is one of the evolutionary computation algorithms. PSO has attracted a lot of attention in recent years because of the following reasons (Parsopoulos & Vrahatis, 2002): First, it requires only a few lines of computer code to realize the PSO algorithm. Second, its search technique using not the gradient information but the values of the objective function makes it an easy-to-use algorithm. Third, it is computationally inexpensive, since its memory and CPU speed requirements are very low. Fourth, it does not require a strong assumption made in conventional deterministic methods such as linearity, differentiability, convexity, separability or non-existence of constraints in order to solve the problem efficiently. Finally, its solution does hardly depend on initial states of particles, which could be a great advantage in engineering design problems based on optimization approaches. Further, Sedlaczek and Eberhard (2006) recently developed an augmented Lagrangian particle swarm optimization (ALPSO) algorithm to handle the optimization problem subject to equality and inequality constraints.

The aim of this paper is to develop a simple and computationally tractable tuning strategy for robust PID controllers satisfying multiple H_∞ specifications. Finding such controller gains is known to be computationally intractable by the conventional techniques. Therefore, in order to solve simply and directly such a design problem without using any complicated manipulations, we first formulate the ALPSO based constrained optimization problem, and then present its distinctive features. It is important to note that a set of PID gains can be directly obtained by solving a non-convex optimization problem based on the ALPSO technique, which is the main difference from the conventional methods by Hwang and Hsiao (2002) and Ho (2003). Also, the proposed technique is applicable both to stable and to unstable systems, which is different from Kristiansson and Lennartson (2006). However, it is difficult to guarantee its effectiveness in a theoretical way, because PSO is essentially a stochastic approach. Therefore, several numerical examples are given to verify the effectiveness of our robust PID controller design technique.

2. Problem formulation

Consider the standard PID feedback control system shown in Fig. 1 where $r(t)$ is the reference signal, $u(t)$ is the control signal, $y(t)$ is the controlled output, $d(t)$ is the disturbance input, and $w(t)$ is the sensor noise. $P(s) = N(s)/D(s)$ is the linear time-invariant system where $N(s)$ and $D(s)$ are coprime

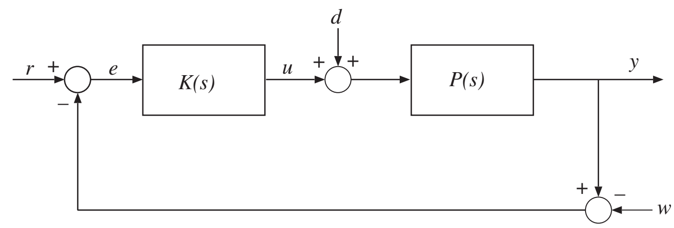

Fig. 1. Block diagram of the PID feedback control system.

polynomials in s defined for $n < m$ as

$$N(s) := a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0,$$
$$D(s) := s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0. \quad (1)$$

$K(s)$ denotes the PID controller which is augmented by a low pass filter on the derivative part of the traditional PID controller and given as

$$K(s) := k_p\left(1 + \frac{1}{t_i s} + \frac{t_d s}{1 + (t_d/N)s}\right), \quad (2)$$

where $k_p \in \mathbb{R}$ is the proportional gain, $t_i \in \mathbb{R}$ is the integral time, $t_d \in \mathbb{R}$ is the derivative time, and t_d/N is the filter time constant. It is assumed that the design parameters (k_p, t_i, t_d, N) are all positive real numbers. Therefore, the feasible parameter domain Δ_f is $\Delta_f := \{(k_p, t_i, t_d, N) \in \mathbb{R}^4 : k_p > 0, t_i > 0, t_d > 0, N > 0\}$.

For the system in Fig. 1, the loop transfer function is $L(s) = P(s)K(s)$, and then sensitivity function $S(s)$ and complementary sensitivity function $T(s)$ are defined as

$$S(s) := 1/(1 + L(s)), \quad T(s) := L(s)/(1 + L(s)). \quad (3)$$

The robust performance criteria considered in this paper, which are based on $S(s)$ and $T(s)$, are given as

$$\sup_{\omega \geqslant 0} |W_S(j\omega)S(j\omega)| \leqslant 1, \quad \sup_{\omega \geqslant 0} |W_T(j\omega)T(j\omega)| \leqslant 1, \quad (4)$$

where $W_S(s)$ and $W_T(s)$ are stable frequency-dependent weighting functions which represent the required stability and performance specifications.

The problem of this paper is to search for the design parameters (k_p^*, t_i^*, t_d^*, N^*) of (k_p, t_i, t_d, N) in PID controller (2), which guarantee that (i) the closed-loop system in Fig. 1 is internally stable, and (ii) the robust performance criteria in (4) are satisfied, based on the optimization approach. Note that the above one is among the fixed structure/order H_∞ controller design problems, which is a challenge to control system engineers since it is still a computationally intractable and highly time-consuming complex problem. Therefore, it is one of the important research issues to develop simple design strategies for optimal robust PID controllers.

Next, the optimization problem to determine the parameters (k_p^*, t_i^*, t_d^*, N^*) of PID controller (2) is explicitly formulated.

Please cite this article as: T.Kim T. -H., et al. Robust PID controller tuning based on the constrained particle swarm optimization. *Automatica* (2007), doi: 10.1016/j.automatica.2007.08.017

In this paper, the following objective function is adopted:

$$\lambda_{\max}(k_p, t_i, t_d, N) := \arg\max_{\lambda_i}\{\mathrm{Re}(\lambda_i(k_p, t_i, t_d, N)), \forall i\}, \tag{5}$$

where $\lambda_i(\cdot) \in \mathbb{C}$ denotes the ith pole of the closed-loop system $T_{\mathrm{cl}}(s)(=T(s))$ in Fig. 1, and $\mathrm{Re}(\lambda_i(\cdot))$ denotes the real part of $\lambda_i(\cdot) \in \mathbb{C}$. Thus, $\lambda_{\max}(k_p, t_i, t_d, N)$ in (5) denotes the maximum one among the real parts of all poles of $T_{\mathrm{cl}}(s)$ with given (k_p, t_i, t_d, N). Then, the optimization problem is summarized as follows:

$$\min_{k_p, t_i, t_d, N} \lambda_{\max}(k_p, t_i, t_d, N) \tag{6}$$

subject to $(k_p, t_i, t_d, N) \in \Delta_f$, the H_∞ performance criteria (4), and

$$\lambda_{\max}(k_p, t_i, t_d, N) \leqslant \varepsilon, \tag{7}$$

where $\varepsilon < 0$ is given by the designer. Note that the constraint (7) is introduced to guarantee the stability of the closed-loop system. Also, the values of four design parameters (k_p, t_i, t_d, N) are directly determined by solving the above optimization problem, not assuming that at least one of them is fixed in advance by the designer as Hwang and Hsiao (2002) and Ho (2003).

3. Robust PID controller design based on the ALPSO algorithm

In this section, a concrete design procedure to determine the design parameters (k_p^*, t_i^*, t_d^*, N^*) by solving the optimization problem considered in Section 2 based on the ALPSO algorithm will be presented. First, a brief overview of ALPSO is presented.

3.1. Constrained PSO scheme

Consider the following optimization problem: for given $f(\boldsymbol{x})$, $\boldsymbol{g}(\boldsymbol{x})$ and $\boldsymbol{h}(\boldsymbol{x})$,

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}), \quad \boldsymbol{x} \in \mathbb{D} \subseteq \mathbb{R}^{n_p} \tag{8}$$

subject to

$$\begin{aligned} \boldsymbol{g}(\boldsymbol{x}) &= \boldsymbol{0}, \quad \boldsymbol{g} : \mathbb{R}^{n_p} \to \mathbb{R}^{m_e}, \\ \boldsymbol{h}(\boldsymbol{x}) &\leqslant \boldsymbol{0}, \quad \boldsymbol{h} : \mathbb{R}^{n_p} \to \mathbb{R}^{m_i}, \end{aligned} \tag{9}$$

where $\mathbb{D}$ denotes the search space. Then, the augmented Lagrange multiplier method is introduced to transform the above constrained optimization problem into an unconstrained optimization problem as

$$L(\boldsymbol{x}, \boldsymbol{\alpha}, \boldsymbol{\beta}) = f(\boldsymbol{x}) + \sum_{\ell=1}^{m_e+m_i} \alpha_\ell \theta_\ell(\boldsymbol{x}) + \sum_{\ell=1}^{m_e+m_i} \beta_\ell \theta_\ell^2(\boldsymbol{x}), \tag{10}$$

where

$$\theta_\ell(\boldsymbol{x}) = \begin{cases} g_\ell(\boldsymbol{x}), & 1 \leqslant \ell \leqslant m_e, \\ \max\left[h_{\ell-m_e}(\boldsymbol{x}), \dfrac{-\alpha_\ell}{2\beta_\ell}\right], & m_e+1 \leqslant \ell \leqslant m_e+m_i \end{cases}$$

and $\boldsymbol{\alpha} := (\alpha_1, \alpha_2, \ldots, \alpha_{m_e+m_i})^{\mathrm{T}} \in \mathbb{R}^{m_e+m_i}$ and $\boldsymbol{\beta} := (\beta_1, \beta_2, \ldots, \beta_{m_e+m_i})^{\mathrm{T}} \in \mathbb{R}^{m_e+m_i}$ denote the Lagrange multiplier and the penalty factors, respectively. The third term in (10) is introduced to guarantee that the solution $\boldsymbol{x}^*$ of (8) subject to (9) is not only a stationary point but also a minimum of $L(\boldsymbol{x}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ in (10) for the correct Lagrange multiplier $\boldsymbol{\alpha}^*$. However, $\boldsymbol{\alpha}^*$ and the appropriate penalty factors $\boldsymbol{\beta}$ are problem dependent and thus unknown.

Hence, $\boldsymbol{x}^*$ is obtained by solving a sequence of $L(\boldsymbol{x}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ with subsequent updates of $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{x}$ as

$$\alpha_\ell^{v+1} = \alpha_\ell^v + 2\beta_\ell^v \theta_\ell(\boldsymbol{x}^v), \tag{11}$$

where $\boldsymbol{\alpha}^0 = \boldsymbol{0}$ and $\boldsymbol{\beta}^0 = \boldsymbol{\beta}_0$ which is given by the designer; for given ε_g and ε_h which denote the tolerances for acceptable constraint violations

$$\beta_\ell^{v+1} = \begin{cases} 2\beta_\ell^v & \text{if } |J_\ell(\boldsymbol{x}^v)| > |J_\ell(\boldsymbol{x}^{v-1})| \text{ and } |J_\ell(\boldsymbol{x}^v)| > \varepsilon_\ell, \\ 0.5\beta_\ell^v & \text{if } |J_\ell(\boldsymbol{x}^v)| \leqslant \varepsilon_\ell, \\ \beta_\ell^v & \text{else}, \end{cases} \tag{12}$$

where $J_\ell(\cdot) = g_\ell(\cdot)$ and $\varepsilon_\ell = \varepsilon_g$ for $1 \leqslant \ell \leqslant m_e$, $J_\ell(\cdot) = h_\ell(\cdot)$ and $\varepsilon_\ell = \varepsilon_h$ for $m_e+1 \leqslant \ell \leqslant m_e+m_i$. Then, $\boldsymbol{x}^v$ in (11)–(12) is updated based on the conventional PSO algorithm (Eberhart & Kennedy, 1995) described as follows: Consider a swarm consisting of m_p particles. The ith particle $\boldsymbol{x}_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,n_p})^{\mathrm{T}} \in \mathbb{R}^{n_p}$ is manipulated according to the following equations:

$$\begin{aligned} \boldsymbol{x}_i^{k+1} &= \boldsymbol{x}_i^k + \boldsymbol{v}_i^{k+1}, \\ \boldsymbol{v}_i^{k+1} &= \omega \boldsymbol{v}_i^k + c_1 \zeta_i^k (\boldsymbol{x}_i^{\mathrm{best},k} - \boldsymbol{x}_i^k) + c_2 \xi_i^k (\boldsymbol{x}_{\mathrm{swarm}}^{\mathrm{best},k} - \boldsymbol{x}_i^k), \end{aligned} \tag{13}$$

where the inertia factor ω, the cognitive scaling factor c_1, and the social scaling factor c_2 influence the particle trajectories and thus the convergence and search diversity properties, which are given by the designer. The random numbers ζ_i^k and ξ_i^k are uniformly distributed in [0, 1] and represent the stochastic behaviors. $\boldsymbol{x}_i^{\mathrm{best},k}$ defined as

$$\boldsymbol{x}_i^{\mathrm{best},k} := \arg\min_{\boldsymbol{x}_i^j}\{L(\boldsymbol{x}_i^j, \boldsymbol{\alpha}^v, \boldsymbol{\beta}^v), 0 \leqslant j \leqslant k\} \tag{14}$$

denotes the best previously obtained position of the ith particle. $\boldsymbol{x}_{\mathrm{swarm}}^{\mathrm{best},k}$ defined as

$$\boldsymbol{x}_{\mathrm{swarm}}^{\mathrm{best},k} := \arg\min_{\boldsymbol{x}_i^k}\{L(\boldsymbol{x}_i^k, \boldsymbol{\alpha}^v, \boldsymbol{\beta}^v), \forall i\} \tag{15}$$

denotes the best position in the entire swarm at the current iteration k. Based on the above algorithm, $\boldsymbol{x}^v$ in (11) and (12) is obtained as $\boldsymbol{x}^v = \boldsymbol{x}_{\mathrm{swarm}}^{\mathrm{best},k_{\max}}$ where $k_{\max}$ is a user-defined maximum iteration number of k. The flowchart of the ALPSO algorithm is given in Fig. 2.

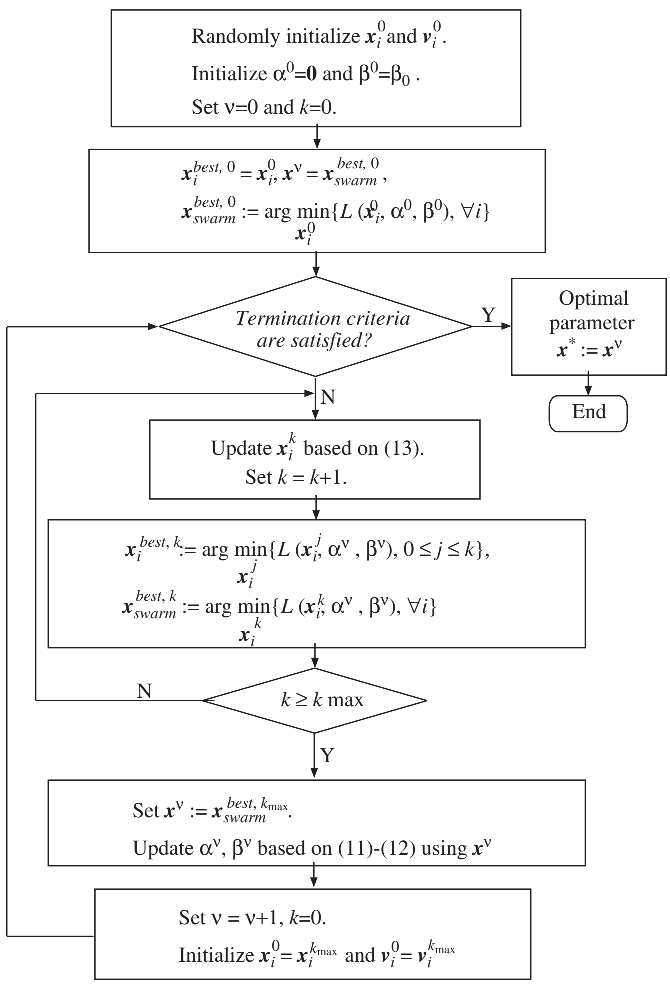

Fig. 2. Flowchart of the ALPSO.

3.2. Robust PID controller tuning scheme

In order to find the parameters $(k_p^*, t_i^*, t_d^*, N^*)$ based on the ALPSO algorithm, we reformulate the optimization problem (6) subject to (4) and (7). First, define each particle containing the design parameters as

$$\begin{aligned} \boldsymbol{x} &:= (x_1, x_2, x_3, x_4)^{\mathrm{T}} \\ &= (\log_{10} k_p, \log_{10} t_i, \log_{10} t_d, \log_{10} N)^{\mathrm{T}}. \end{aligned} \tag{16}$$

Note that the technique using common logarithms as in (16) enables one to search a broader parameter space of $(k_p,\ t_i,\ t_d,\ N)$. In case that the PID controller has some negative gains, we set $\boldsymbol{x} := (k_p, t_i, t_d, N)$. Then, the PID controller in (2) is modified based on (16) as

$$K(s; \boldsymbol{x}) = 10^{x_1}\left(1 + \frac{1}{10^{x_2}s} + \frac{10^{x_3}s}{1 + 10^{(x_3-x_4)}s}\right). \tag{17}$$

Also, the objective function (5) is written as

$$\lambda_{\max}(\boldsymbol{x}) := \arg\max_{\lambda_i(\boldsymbol{x})}\{\mathrm{Re}(\lambda_i(\boldsymbol{x})), \forall i\}, \tag{18}$$

where $\lambda_i(\boldsymbol{x}) \in \mathbb{C}$ denotes the ith pole of the closed-loop system $T_{\mathrm{cl}}(s;\boldsymbol{x}) = L(s;\boldsymbol{x})/(1+L(s;\boldsymbol{x}))$ where $L(s;\boldsymbol{x}) := P(s)K(s;\boldsymbol{x})$. The optimization problem to find a set of PID controller gains $(k_p^*,\ t_i^*,\ t_d^*,\ N^*)$ is summarized as:

Optimization problem for ALPSO.

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}) := \lambda_{\max}(\boldsymbol{x}), \quad \boldsymbol{x} \in \mathbb{D} \subseteq \mathbb{R}^4 \tag{19}$$

subject to

$$\begin{aligned} \boldsymbol{h}(\boldsymbol{x}) &:= (h_1(\boldsymbol{x}), h_2(\boldsymbol{x}), h_3(\boldsymbol{x}))^{\mathrm{T}} \\ &= \begin{bmatrix} \sup_{\omega \geqslant 0} |W_S(\mathrm{j}\omega)S(\mathrm{j}\omega; \boldsymbol{x})| - 1 \\ \sup_{\omega \geqslant 0} |W_T(\mathrm{j}\omega)T(\mathrm{j}\omega; \boldsymbol{x})| - 1 \\ \lambda_{\max}(\boldsymbol{x}) - \varepsilon \end{bmatrix} \leqslant \mathbf{0}, \end{aligned} \tag{20}$$

where $S(\mathrm{j}\omega;\boldsymbol{x}) := 1/(1 + L(\mathrm{j}\omega;\boldsymbol{x}))$ and $T(\mathrm{j}\omega;\boldsymbol{x}) := L(\mathrm{j}\omega;\boldsymbol{x})/(1 + L(\mathrm{j}\omega;\boldsymbol{x}))$.

The search space is set as $\mathbb{D} := \{\boldsymbol{x} \in \mathbb{R}^4 : \underline{\boldsymbol{x}} \leqslant \boldsymbol{x} \leqslant \overline{\boldsymbol{x}}\}$ with $\overline{\boldsymbol{x}} > \underline{\boldsymbol{x}}$ which are given by the designer. It may be useful in applications where the presence of high-frequency noise imposes a limit on the maximum allowable derivative action as shown in Grassi and Tsakalis (2000). Also, the solution of the above optimization problem for ALPSO does hardly depend on initial states of particles (initial choice of controller gains), which could be a great advantage in engineering design problems based on optimization approaches. Further, there is no necessity for fixing one of the controller gains in advance, and a set of PID parameters satisfying the given constraints can be simply obtained by solving the constrained optimization problem (19)–(20) using the ALPSO algorithm. Therefore, the above features enable one to overcome some drawbacks of the conventional methods by Åström et al. (1998), Hwang and Hsiao (2002) and Ho (2003) mentioned in Section 1.

In the following section, the distinctive features of our robust PID controller tuning strategy are verified. Specifically, in Sections 4.2 and 4.3, it will be shown that the parameters of PID controller which has different structure from (2) can also be found easily in a similar way to our ALPSO based parameter tuning scheme.

4. Simulation examples

4.1. Example 1

Consider the simple magnetic levitation system given in Sugie, Simizu, and Imura (1993). The linearized model about an equilibrium point of $y = 0.018$ m is given as $P(s) = 7.147/((s - 22.55)(s + 20.9)(s + 13.99))$. Each particle in the swarm for ALPSO and the PID controller are set as (16) and (17), respectively. The frequency-dependent weighting functions $W_S(s)$ and $W_T(s)$ in (20) are, respectively, given as

$$\begin{aligned} W_S(s) &= 5/(s + 0.1), \\ W_T(s) &= 4.3867 \times 10^{-7}(s + 0.066) \\ &\quad \times (s + 31.4)(s + 88)(10^4/(s + 10^4))^2. \end{aligned} \tag{21}$$

The search space is set as $\mathbb{D} := \{\boldsymbol{x} \in \mathbb{R}^4 : (2, -1, -1, 1)^{\mathrm{T}} \leqslant \boldsymbol{x} \leqslant (4, 1, 1, 3)^{\mathrm{T}}\}$. The swarm size is equal to $m_p = 100$, $k_{\max} = 3$,

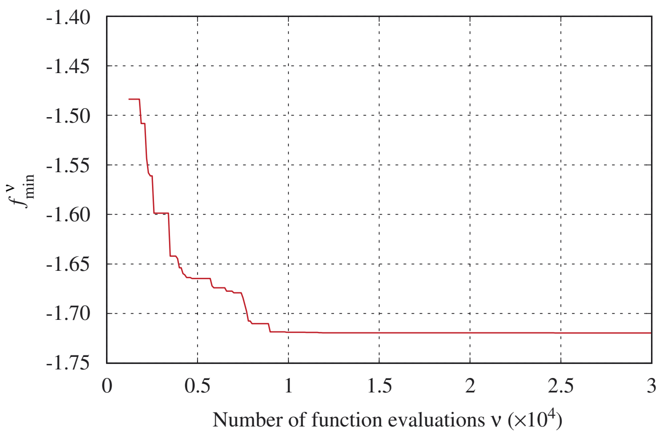

Fig. 3. Plot of the objective function value $f(\boldsymbol{x}^{v})$ during the optimization process.

$\boldsymbol{v}_i^0 = \boldsymbol{0}$, $\boldsymbol{\alpha}^0 = \boldsymbol{0}$, $\boldsymbol{\beta}^0 = [1, 1, \ldots, 1]$, and $\omega = c_1 = c_2 = 0.9$. In (12), ε_h is set as $\varepsilon_h = 10^{-4}$. Then, the optimization problem (19) subject to (20) is solved based on the ALPSO algorithm given in Section 3. The plot of $f^{v}_{\min} := \min\{f(\boldsymbol{x}^{v}_{j}),\ j \in (1, 100)\}$ where $\boldsymbol{x}^{v}_{j}$ denotes the particle satisfying the constraints (20) is illustrated in Fig. 3. There is no $\boldsymbol{x}^{v}_{j}$ satisfying the given constraints for about $v < 0.12 \times 10^4$, and $f^{v}_{\min}$ converges to -1.7197 $(=\lambda_{\max}(\boldsymbol{x}^*))$ after about $v = 2.5 \times 10^4$. The computation time until $v = 2.5 \times 10^4$ is about 687 s. The optimal value $\boldsymbol{x}^*$ of $\boldsymbol{x}$ is obtained as $\boldsymbol{x}^* = (3.2548, -0.8424, -0.7501, 2.3137)^{\mathrm{T}}$. Therefore, the PID controller is designed from $\boldsymbol{x}^*$ as

$$K(s) = 1798.1\left(1 + \frac{1}{0.1438s} + \frac{0.1778s}{1 + (8.6336 \times 10^{-4})s}\right). \tag{22}$$

The resulting sensitivity and complementary sensitivity functions are shown in Fig. 4, which demonstrates that the given constraints in (20) are satisfied.

4.2. Example 2

Consider the system in Ho (2003), where the plant and the PID controller are, respectively, given as follows:

$$P(s) = (s-1)/(s^2 + 0.8s - 0.2), \tag{23}$$

$$K(s; \boldsymbol{x}) = x_1 + x_2/s + x_3 s, \tag{24}$$

where $\boldsymbol{x} := (x_1, x_2, x_3) = (k_p, k_i, k_d) \in \mathbb{R}^3$ denotes the each particle in the swarm. In Ho (2003), the constraint only on the complementary sensitivity function is considered as $\|W_T(s)T(s;\boldsymbol{x})\|_\infty \leqslant 1$ where $W_T(s) = (s+0.1)/(s+1)$. For the above system, we apply the ALPSO technique to find the optimal gains $\boldsymbol{x}^* = (k_p^*, k_i^*, k_d^*)$. Here, an additional constraint such as $\lambda_{\max}(\boldsymbol{x}) \leqslant 0$ is introduced to guarantee the closed-loop system stability. All design parameters are set identically to those of Section 4.1. The search space is $\mathbb{D} := \{\boldsymbol{x} \in \mathbb{R}^3 : (-1.8, -1, -1)^{\mathrm{T}} \leqslant \boldsymbol{x} \leqslant (-0.2, 1, 1)^{\mathrm{T}}\}$. Note that Ho (2003) requires an additional task, root locus ideas (Datta, Ho, & Bhattacharyya, 2000), to find a

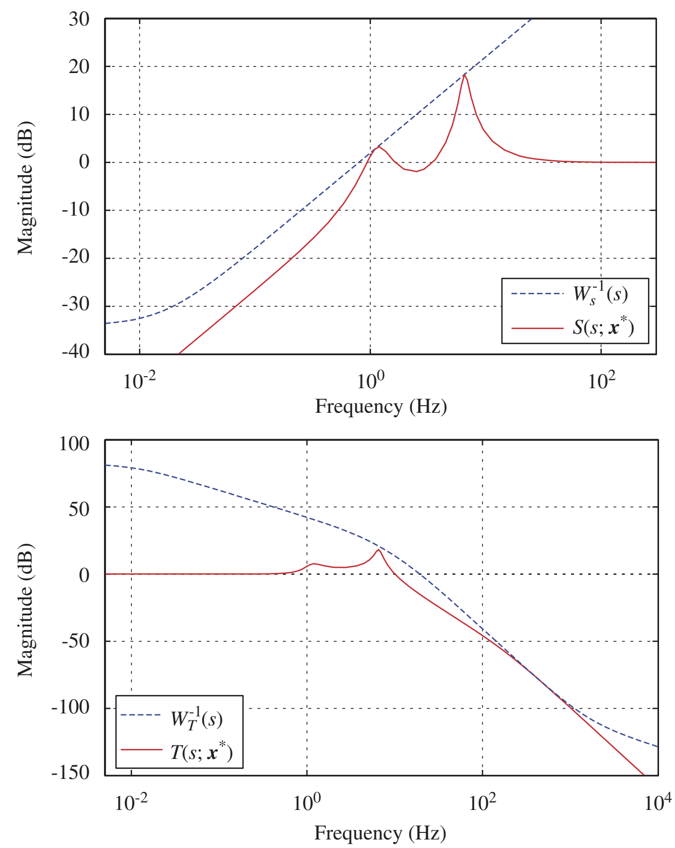

Fig. 4. Bode plots of sensitivity function $S(s;\boldsymbol{x}^*)$ and complementary sensitivity function $T(s;\boldsymbol{x}^*)$.

necessary condition $k_p \in (-1.8, -0.2)$ for the existence of stabilizing gains k_i and k_d. Then, based on $k_p \in (-1.8, -0.2)$, the admissible sets of (k_p, k_i, k_d) satisfying $\|W_T(s)T(s;\boldsymbol{x})\|_\infty \leqslant 1$ are obtained. However, it is not necessary to find in advance the feasible region of k_p using an additional method in our approach. The optimal gains are found as $\boldsymbol{x}^* = (k_p^*, k_i^*, k_d^*) = (-0.5545, -0.04513, -0.4615)$ for which $f(\boldsymbol{x}^*) = \lambda_{\max}(\boldsymbol{x}^*) = -0.4376$, and $\|W_T(s)T(s;\boldsymbol{x}^*)\|_\infty \leqslant 1$ is guaranteed. Note that the PID controller gains are found by just solving the constrained optimization problem based on ALPSO, which shows the simplicity and effectiveness of our approach comparing to some rather complicated procedures of Ho (2003).

4.3. Example 3

Consider a mixed sensitivity control problem in Fig. 5 presented in Saeki (2006), where

$$P(s) = \begin{bmatrix} 1/(s+1) & 0.2/(s+3) \\ 0.1/(s+2) & 1/(s+1) \end{bmatrix}, \tag{25}$$

$V(s) = v_1(s)I_2$, $W(s) = v_2(s)I_2$, $v_1(s) = (s+3)/(3s+0.3)$, $v_2(s) = (10s+2)/(s+40)$, and $a = 0.01$. For the above system, the design problem is to develop a decentralized PID controller

Please cite this article as: T.Kim T. -H., et al. Robust PID controller tuning based on the constrained particle swarm optimization. *Automatica* (2007), doi: 10.1016/j.automatica.2007.08.017

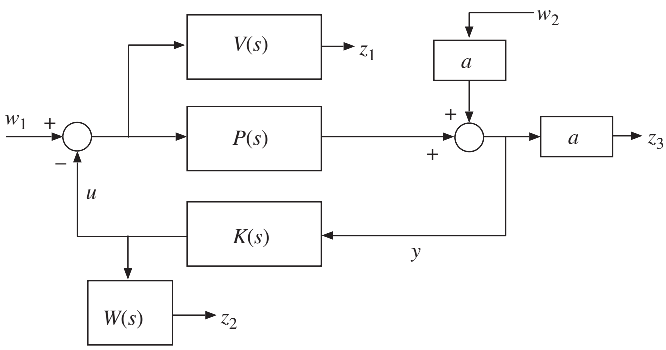

Fig. 5. Mixed sensitivity control problem.

having the following structure:

$$K(s) = \begin{bmatrix} k_{p_1} & 0 \\ 0 & k_{p_2} \end{bmatrix} + \begin{bmatrix} k_{i_1} & 0 \\ 0 & k_{i_2} \end{bmatrix} \frac{1}{s} + \begin{bmatrix} k_{d_1} & 0 \\ 0 & k_{d_2} \end{bmatrix} \frac{s}{1+0.01s} \tag{26}$$

which should satisfy $\|T_{zw}(s)\|_\infty \leqslant 1$ where $T_{zw}(s)$ denotes the closed-loop transfer function from w to z. Let $\boldsymbol{x} := (x_1, x_2, x_3, x_4, x_5, x_6)^{\mathrm{T}} = (\log_{10} k_{p1}, \log_{10} k_{i_1}, \log_{10} k_{d_1}, \log_{10} k_{p2}, \log_{10} k_{i_2}, \log_{10} k_{d_2})^{\mathrm{T}}$ denote the each particle in the swarm. The objective function $f(\boldsymbol{x})$ is defined as $f(\boldsymbol{x}) = \|T_{zw}(s;\boldsymbol{x})\|_\infty$, and the constraint $h(\boldsymbol{x}) := \lambda_{\max}(\boldsymbol{x}) < \varepsilon = -10^{-4}$ is introduced to guarantee the stability of the system in Fig. 5. The swarm size is equal to $m_p = 100$, $k_{\max} = 3$, $\boldsymbol{v}_i^0 = \boldsymbol{0}$, $\boldsymbol{\alpha}^0 = \boldsymbol{0}$, $\boldsymbol{\beta}^0 = [1, 1, \dots, 1]$, $\omega = 0.9$, $c_1 = c_2 = 0.8$, and $\varepsilon_h = 10^{-4}$. The search space is set as $\mathbb{D} := \{\boldsymbol{x} \in \mathbb{R}^6 : \underline{\boldsymbol{x}} \leqslant \boldsymbol{x} \leqslant \bar{\boldsymbol{x}}\}$ where $\underline{\boldsymbol{x}} := -(3, 3, \dots, 3) \in \mathbb{R}^6$ and $\bar{\boldsymbol{x}} := (3, 3, \dots, 3) \in \mathbb{R}^6$. Then, the optimization process based on the ALPSO algorithm is performed, and the following controller gains are obtained: $(k_{p1}^*, k_{i_1}^*, k_{d_1}^*, k_{p2}^*, k_{i_2}^*, k_{d_2}^*) = (1.8015, 1.9477, 2.6827 \times 10^{-2}, 1.8252, 1.8135, 1.188 \times 10^{-2})$. In this case, the value $f(\boldsymbol{x}^*) = \|T_{zw}(s;\boldsymbol{x}^*)\|_\infty$ is 0.5842. Note that $f(\boldsymbol{x}^*) = 0.5842$ of our approach is not worse than the result, $f(\boldsymbol{x}^*) = 0.5882$, of Saeki (2006). Also, the computation time is similar to that of Saeki (2006). However, it should be noted that the controller gains are found by just solving the constrained optimization problem based on the ALPSO technique. It means that our strategy is considerably simple, and thus is easily implementable in various engineering applications using PID controller, since it enables one to avoid specific LMI transformations given in Saeki (2006).

5. Conclusion

In this paper, a considerably simple and computationally tractable tuning strategy for robust PID controllers satisfying multiple H_∞ specifications is developed. Generally, the design problem for optimal robust PID controllers based on H_∞ techniques results in a non-convex optimization problem subject to multiple inequality constraints. In order to solve simply and directly such a design problem, the ALPSO based robust gain tuning scheme for PID controllers is proposed. It performs without any conservative assumption required in the conventional methods, and further enables one to find a set of PID gains directly by just solving the constrained optimization problem in a straightforward way. It is demonstrated by several simulation examples that the proposed approach is considerably simple and convenient to use. From the above observations, it is expected that the results of this paper will contribute to the development of practical PID controllers used in various engineering applications. On the other hand, since PSO used in this paper is essentially a stochastic approach, it may fail to find PID gains even if the constraints are feasible. Thus, future research will be devoted to improve the convergence performance and the search ability of constrained PSO algorithm.

References

Åström, K. J., Panagopoulos, H., & Hägglund, T. (1998). Design of PI controllers based on non-convex optimization. *Automatica*, *34*(5), 585–601.

Datta, A., Ho, M. T., & Bhattacharyya, S. P. (2000). *Structure and synthesis of PID controllers*. London: Springer.

Eberhart, R. C., & Kennedy, J. (1995). A new optimizer using particle swarm theory. In *Proceedings of the sixth international symposium on micromachine and human science* (Vol. 1, pp. 39–43).

Grassi, E., & Tsakalis, K. (2000). PID controller tuning by frequency loop-shaping: Application to diffusion furnace temperature control. *IEEE Transactions on Control Systems Technology*, *8*(5), 842–847.

Ho, M. (2003). Synthesis of H_∞ PID controllers: A parametric approach. *Automatica*, *39*(6), 1069–1075.

Ho, M., & Lin, C. (2003). PID controller design for robust performance. *IEEE Transactions on Automatic Control*, *48*(8), 1404–1409.

Hwang, C., & Hsiao, C.-Y. (2002). Solution of a non-convex optimization arising in PI/PID control design. *Automatica*, *38*(11), 1895–1904.

Kristiansson, B., & Lennartson, B. (2006). Robust tuning of PI and PID controllers: Using derivative action despite sensor noise. *IEEE Control Systems Magazine*, *26*(1), 55–69.

Parsopoulos, K. E., & Vrahatis, M. N. (2002). Recent approaches to global optimization problems through particle swarm optimization. *Natural Computing*, *1*(2–3), 235–306.

Saeki, M. (2006). Fixed structure PID controller design for standard H_∞ control problem. *Automatica*, *42*(1), 93–100.

Sedlaczek, K., & Eberhard, P. (2006). Using augmented Lagrangian particle swarm optimization for constrained problems in engineering. *Structural and Multidisciplinary Optimization*, *32*(4), 277–286.

Sugie, T., Simizu, K., & Imura, J. (1993). H_∞ control with exact linearization and its application to magnetic levitation systems. In *IFAC 12th World congress* (Vol. 4, pp. 363–366), July 1993.

Please cite this article as: T.Kim T. -H., et al. Robust PID controller tuning based on the constrained particle swarm optimization. *Automatica* (2007), doi: 10.1016/j.automatica.2007.08.017

Tae-Hyoung Kim received the B.S. and M.S. degrees in mechanical engineering from Chung-Ang University, Korea, in 1999 and 2001, respectively. He received the Ph.D. degree in informatics from Kyoto University, Japan, in 2006. From April 2006 to February 2007, he was a guest research associate in the Department of Systems Science, Kyoto University. He is currently a researcher in the Japan Science and Technology Agency (JST). His current research interests include model predictive control, iterative learning control, cooperative control, particle swarm optimization and system identification.

Ichiro Maruta received the B.S. degree in engineering from Kyoto University, Japan, in 2006. He is currently a graduate student in the Department of Systems Science, Kyoto University. His current research interests include fixed-order control, numerical optimization and discrete-valued control.

Toshiharu Sugie received the B.E., M.E., and Ph.D. degrees in engineering from Kyoto University, Japan, in 1976, 1978 and 1985, respectively. From 1978 to 1980, he was a research member of Musashino Electric Communication Laboratory in NTT, Musashino, Japan. From 1984 to 1988, he was a research associate of Department of Mechanical Engineering, University of Osaka Prefecture, Osaka. In 1988, he joined Kyoto University, where he is currently a professor of Department of Systems Science. His research interests are in robust control, learning control, nonlinear control, identification for control, and their application to mechanical systems. He is a fellow of the IEEE.

Please cite this article as: T.Kim T. -H., et al. Robust PID controller tuning based on the constrained particle swarm optimization. *Automatica* (2007), doi: 10.1016/j.automatica.2007.08.017