Comparison of engineering methods of loss prediction in thin ferromagnetic laminations

Sergey E. Zirka a, Yury I. Moroz a, Philip Marketos b,*, Anthony J. Moses b

a Department of Physics and Technology, the Dnepropetrovsk National University, Ukraine, 49050, Dnepropetrovsk, Naukova str. 13
b Wolfson Centre for Magnetics, School of Engineering, Cardiff University, Cardiff, The Parade, CF24 3AA, UK

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Abstract

The loss predictive methods based on the static and dynamic components of power loss are compared with the methods where the total loss is subdivided into hysteresis, classical and excess components. It is explained why the simplest two-component methods can be preferable in some cases. An approach to the characterization of a given steel is outlined.

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1. Introduction

An important practical challenge in the design of electrical machines and devices is to develop methods of predicting power loss in soft magnetic laminations under arbitrary waveform of the periodic magnetic induction. Although the loss can be predicted quite accurately by means of finite-element [1] or finite-difference (FD) [2] solvers of appropriate Maxwell equations, the simplified engineering methods applicable to a bulk material remain of considerable significance, and new predictive techniques appear periodically in the literature. The term engineering means that the methods can be implemented on a calculator or by means of library procedures available in most mathematical packages.

Input data for all existing methods are loss values measured under sinusoidal flux densities. These techniques can be broadly subdivided into two-component and three-component methods according to the way in which this data is used. In the two-component methods the measured loss \( W \) is subdivided into hysteresis (static) and eddy current (dynamic) losses [3, 4, 5]. Three-component methods rely on the full loss separation which additionally supposes the subdivision of the dynamic loss, \( W_{\text{dyn}} \), into classical, \( W_{\text{clas}} \), and excess loss, \( W_{\text{exc}} \) [6, 7]. The latter necessitates a formula representation of \( W_{\text{clas}} \) and \( W_{\text{exc}} \), which requires the use of the material resistivity \( \rho \) and lamination thickness \( d \). The possibilities and limits of the methods above in predicting dynamic loss are discussed in this paper. To avoid the inaccuracies caused by minor hysteresis loops, the same static hysteresis model [8] is used to evaluate hysteresis loss, \( W_{\text{h}} \), in testing the methods. The choice of this history-dependent model is caused by its ability to reproduce exactly experimental major loop and first-order reversal curves.

2. Eddy current loss

Regardless of the method employed, the first point to be considered is obtaining the eddy current loss under arbitrary flux density, \( W_{\text{ec}} \), proceeding from the value, \( W_{\text{ec}}^{\text{sin}} \), which occurs in sinusoidal regime. If both regimes are characterized by the same peak induction \( B_{\text{m}} \), then, according to [3] and regardless of the magnetization frequency,

\[
W_{\text{ec}}(B_{\text{m}}) = L_{\text{e}} \cdot W_{\text{ec}}^{\text{sin}}(B_{\text{m}}) \tag{1}
\]

where the loss factor, \( L_{\text{e}} \), is determined from the magnitudes \( B_{i} \) of harmonics of nonsinusoidal induction.
Another way of evaluating \( W_{ec} \) uses the form factor coefficient (FFC) \( F_c \) of the magnetization voltage, which is the ratio of the form factor of the nonsinusoidal voltage to that of a sine voltage [7]. It is assumed [7] that the same magnitude magnetization curve. A well-known example is the regime [3], a period, i.e. minor hysteresis loops can appear in the \( (BV) \) which is different for different fundamental harmonic of the magnitude \( m \) takes

\[
W_{ec}(B_m) = F_c^2 \cdot W_{ec}^{\sin}(B_m). \tag{3}
\]

As seen from (1) and (3) these formulae differ only in the multipliers of \( W_{ec}^{\sin}(B_m) \). So the difference between (1) and (3) can be shown by comparing \( L_f \) and \( F_c^2 \) in a regime where the induction waveform can have more than two peaks during a period, i.e. minor hysteresis loops can appear in the magnetization curve. A well-known example is the regime [3], [6] when the induction waveform includes a third harmonic of magnitude \( B_3 \) phase shifted by an angle \( \phi_3 \) with respect to the fundamental harmonic of the magnitude \( B_1 \). The minor loops appear when the ratio \( B_3 / B_1 \) exceeds some boundary value (BV) which is different for different \( \phi_3 \). Calculated values of \( L_f \) and \( F_c^2 \) versus the ratio \( B_3 / B_1 \) are shown in Fig. 1 where points BV separate the regimes with and without minor loops.

![Fig. 1. Values of \( L_f \) and \( F_c^2 \) versus \( B_3 / B_1 \) for \( \phi_3=0 \) and \( \phi_3=180^\circ \).](image)

It is notable that in the absence of minor loops the values of \( L_f \) and \( F_c^2 \) coincide, whereas to the right of points BV the solid and dashed curves in Fig. 1 quickly diverge showing that the FFC becomes useless soon after minor loops appear due to the growth of \( B_3 \). This conclusion is corroborated by comparing the curves in Fig. 1 with the experimental loss curves in [3] and will also be illustrated in section 4 by calculating the loss through the magnetodynamic model (MDM) [2], which is a FD solver of the penetration equation based on the dynamic hysteresis model.

It should be recalled that in two-component methods \( W_{ec}^{\sin} \) is found experimentally (\( W_{ec}^{\sin} = W - W_h \)), whereas in the three-component methods this loss (in J/m³) is identified with the classical loss and calculated as

\[
W_{clas}^{\sin} = \frac{d^2 \pi^2 B_m^2}{6 \rho} f. \tag{4}
\]

Although (4) is only valid at low enough frequencies, it is often used over an unlimited frequency range [6, 7, 9] which can make a three-component method less accurate than the two-component method.

### 3. Excess loss evaluation

A convenient tool to include excess loss in the total loss evaluation is the thin sheet model (TSM) [10] which enables the total loss (\( W = W_h + W_{clas} + W_{exc} \)) to be calculated as

\[
W = \int H_h(B) dB + \int \frac{d^2}{12 \rho} \left( \frac{dB}{dt} \right)^2 dB + \int \delta g(B) \left( \frac{dB}{dt} \right)^{1/\alpha} dB. \tag{5}
\]

The integrands in the first, second and third loss terms of (5) are the inverse hysteresis relationship, \( H_h(B) \), the classical field [11], and the excess field, where \( \delta = 1 \) is a directional parameter. The material function \( g(B) \) controls the shape of the dynamic loop, and the exponent \( \alpha \) determines the frequency law of the excess loss. It has been shown in [12] that \( W_{exc} \sim f^{1/\alpha} \). This means that when \( g(B) = const \) and \( \alpha=2 \), formula (5) can be reduced to the expression [9]

\[
W = W_h + \frac{d^2}{12 \rho} \int_0^\infty \left( \frac{dB}{dt} \right)^2 dt + C_0 \int_0^\infty \left( \frac{dB}{dt} \right)^{3/2} dt \tag{6}
\]

where \( C_0 \) is a fitting parameter. Under sinusoidal induction, (6) is further simplified to give a widely used formula

\[
W = W_h(B_m) + \frac{d^2 \pi^2 B_m^2}{6 \rho} f + C B_m^{1.5} f^{0.5} \tag{7}
\]

where constant \( C \) in the last (excess loss) term is a fitting parameter determined individually for each \( B_m \) and linked with \( C_0 \) in (6) by a multiplier dependent on the induction waveform.

### 4. Comparison of two- and three-component methods

The three-component methods have been proposed to account for different frequency dependencies of classical and excess loss components seen in (7). It was guessed in [6] that the methods based on (7), which take account of this feature, are more accurate than two-component methods built upon the “classical” formulae

\[
W = W_h + k \frac{d^2 \pi^2 B_m^2}{6 \rho} f \tag{8}
\]

or

\[
W = W_h + k \frac{d^2}{12 \rho} \int_0^\infty \left( \frac{dB}{dt} \right)^2 dt. \tag{9}
\]

The introduction of the empirical constant \( k \) in (8) and (9) is caused by the necessity to compensate for the absence of the
excess loss term in these formulae. It is considered that the value of $k$ chosen for the sinusoidal regime through (8) can then be used in (9) employed under arbitrary flux density with the same $B_{in}$.

The difference between the methods based on (6), (7) and (8), (9) can be illustrated from studies of two non-oriented electrical steels differing in the dynamic loss contribution [2]. The first material, Steel-0.1, is 0.1 mm thick, 5.5% Si, $\rho=0.735$ $\mu\Omega\cdot$m, while the other, Steel-0.5, is 0.5 mm thick, 1.8% Si, $\rho=0.432$ $\mu\Omega\cdot$m. It was shown in [10] that the TSM applied to high silicon Steel-0.1 provides the same accurate prediction of the dynamic loops and losses (up to 1 kHz and $B_{in}=1.3$ T) as the MDM [2]. The measured loss dependence shown in Fig. 2 ($B_{in}=1.3$ T) is indistinguishable from that calculated through the TSM (5) with $\alpha=3/2$ ($W_{exc} \approx f^{2/3}$). It can be seen from Fig. 2 that, for example, at $B_{in}=1.3$ T and $f=400$ Hz the excess loss is four times larger than the classical loss, so the total loss obtained through the three-component TSM is expected to be much more accurate than that calculated with (9) if both the models are used in the nonsinusoidal regime. Following [6] and [7], the models have been compared in the regime with a third harmonic whose amplitude reached 30% of the fundamental harmonic. The TSM-calculated dynamic loss is on average 12% more accurate than that evaluated through (9) where $k=5.134$. The reason that the accuracy of the total loss prediction increases only by 3 to 5% is that dynamic loss does not exceed 25% of the total loss. A similar situation has been observed for amorphous material [6] where a 20% improvement in the predicted dynamic loss results in only 5% higher accuracy of the total loss.

![Fig. 2. Measured total losses in Steel-0.1 and Steel-0.5 (solid lines). Dotted, dashed and dash-dotted lines are loss components.](image)

It is interesting that for the thicker Steel-0.5 the three-component methods employed in the conventional manner [6], [7] can give worse results than simple two-component methods. The reason is rooted in the neglect of the complex frequency dependence of the eddy current loss [13] and in the use of low-frequency formula (4) at higher frequencies. Although (4) is valid only under uniform flux (no skin effect) [9], [11], no attention is usually paid to this restriction, and (4) is always used to calculate classical loss. The imperfection of this approach causes the difference between curves 1 and 2 in Fig. 2, each of which is a sum of hysteresis and eddy currents losses. When constructing curve 2 eddy current loss was calculated using (4), whereas curve 1 has been obtained through the MDM which reproduces quite accurately spatially inhomogeneous induction (and thus eddy current loss) over the sheet cross section. This means that the gap between curves 1 and 2 in Fig. 2 is the absolute error of both formula (4) and the excess loss calculated through (4) and (7). As in Steel-0.5 $W_{class} > W_{exc}$ then 17% relative error of $W_{class}$ at say $f=400$ Hz, $B_{in}=1.5$ T leads to 130% relative error of $W_{exc}$. The presence of this error makes parameter $C$ in (7) dependent not only on $B_{in}$ but also on the test frequency $f$. The loss dependence shown by the dotted line (7) in Fig. 2 has been constructed by means of (7) whose parameter $C$ was chosen so that point 2 of this dependence coincides with the measured loss at $B_{in}=1.5$ T, $f=50$ Hz. The deviation of curve (7) from the experimental loss curve demonstrates the inaccuracy of the extrapolation technique [6], [7], according to which parameter $C$ once adjusted at frequency $f$ can then be employed at any new frequency $f_{new}$. The increase of the deviation with frequency shows that the degree of the inaccuracy depends on the difference between $f$ and $f_{new}$ and therefore on the harmonic content of the induction waveform.

To analyze the mentioned regime with third harmonic using (7), the second and third terms in (7) should be multiplied by $F_{c}^2$ and $F_{c}$, respectively [7]:

$$W = W_h + W_{class} + W_{exc} F_c^2 + W_{exc} F_c.$$  (10)

This “general formula” and corresponding loss curve (10) are given in Fig. 3. The use of $F_{c}^2$ as multiplier of the whole dynamic loss (its value $W_{dyn}^2$ was also found at 50 Hz) is illustrated in Fig. 3 by curve (11) obtained through formula

$$W = W_h + W_{dyn} F_c^2.$$  (11)

As expected from the analysis above, both the losses calculated through the FFC become substantially less than the loss evaluated by the MDM (solid line MDM in Fig. 3) when minor hysteresis loops appear at $B_3/B_1>1.1$.

![Fig. 3 Total loss in Steel-0.5 predicted at $B_{in}=1.5$ T with different engineering models and evaluated through the MDM [2].](image)
Taking into account both the equivalence between $F^2_c$ and $L_f$ in the sinusoidal regime, and the advantage of $L_f$ under nonsinusoidal induction, it is reasonable to modify the loss formulae (10) and (11) into

$$W = W_h + W^\text{sin}_{\text{class}} L_f + W^\text{sin}_{\text{exc}} f^{0.5}$$

(12)

and

$$W = W_h + L_f W^\text{sin}_{\text{dyn}} f$$

(13)

These formulae and corresponding loss curves (12) and (13) are given in Fig. 3, which confirms the advantage of using loss factor $L_f$ instead of $F^2_c$ in the case of minor loops.

Another remarkable feature in Fig. 3 is that two-component formula (13) produces the loss curve, which is closer to the reference curve MDM than that calculated by (12). The inferior result obtained through (12) is caused by the excessive decline of curve (7) in Fig. 2. To explain the better accuracy of (13) we should first note that the gap between the measured loss curve and curve 2 in Fig. 2 increases with $f$ almost linearly. This suggests that corresponding excess loss component should also be linear function of $f$, instead of growing as $f^{0.5}$. This is easily verified by using (5) in the sinusoidal regime. The total loss values calculated with $\alpha=1$ ($W_{\text{exc}} f$) lie on the experimental loss curve. Since both classical and excess losses increase in accordance with the same linear law, these components can be combined into one dynamic loss and represented by the last term in (9) where $k=1.407$ when $B_m=1.5$ T.

It should be noted here that $\alpha=1$ and thus $W_{\text{exc}} f$ has been arrived at by extending the low-frequency formula (4) to higher frequencies. The inadequacy of such loss separation was pointed out previously [14]. Its artificial nature is also illustrated in Fig. 4 where the “excess losses”, calculated classically ($W_{\text{exc}}^\text{class} = W - W_{\text{class}}^\text{class}$) for some $B_m$ are plotted versus $f^{0.5}$. These nonlinear dependencies contrast with relations $W_{\text{exc}}^\text{MDM} (\sqrt{f})$ evaluated through the MDM (their nearly linear character is explained by $\alpha=2$ in the MDM).

The fact that $W_{\text{exc}}^\text{class}$ can be less than $W_{\text{exc}}^\text{MDM}$ at low $B_m$ means that the “excess loss” calculated by the classical subtraction technique can be negative [13] thus contradicting its physical meaning. In this situation the simplest two-component methods [4], [5] may be preferable. It does not mean that the excess loss techniques should be discarded; they should be simply applied with care. The Maxwell solver of the type of the MDM [2] might turn out to be the only tool to perform the loss separation sufficiently accurately. This leads to the idea of measuring the total loss of a given steel over a wide range of sinusoidal flux densities and frequencies and using the MDM to split these losses into three components. The results of such characterization and the way in which this data should be used provide a basis for an accurate loss prediction under arbitrary induction/voltage waveform.

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References