Modeling of hysteresis in magnetic cores with frequency-dependent losses

H.G. Brachtendorf*, R. Laur

Institute for Electromagnetic Theory and Microelectronics, University of Bremen, 28334 Bremen, Germany

Received 2 April 1997

Abstract

A novel hysteresis model is presented which exhibits all main features of hysteresis, such as initial magnetization, saturation, coercivity, remanence and frequency-dependent losses. It consists merely of three differential equations with six parameters. Depending on the slope of the outer hysteresis loop four variants of the model are discussed. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Hysteresis; Core loss; Modeling; Simulation

1. Introduction

Transformers and inductors differ from the ideal models incorporated in SPICE2 [1] due to saturation and power losses. Therefore, accurate and reliable hysteresis models are mandatory for an accurate design of circuits including magnetic cores.

Several attempts have been proposed in the last years for the simulation of hysteresis phenomena [2–18]. Some of them were implemented in commercial circuit simulators [2,3,5–7,10–13]. In Refs. [2–12] hysteresis is described by a system of ordinary differential-algebraic equations and in Refs. [13–15] (piecewise) hyperbolic functions have been used. Especially, the model of Jiles and Atherton (JA model) [2–4] which is based on recognized theories of ferromagnetic hysteresis has found much attention. Nevertheless, the modeling of minor loops is not accurate enough. Several improvements of this model exist concerning minor loop behavior and frequency dependence, e.g. Refs. [5–8,18]. In Ref. [5] frequency dependence has been included by a linear differential equation of second order in time for the magnetization $M$ and in Ref. [6], a first-order differential equation for the magnetic field strength $H$ has been introduced. The authors proposed in Ref. [18] another variant of the JA model where the pinning constant of this model has been described by a linear differential equation in time. At low frequencies this model reverts back to the well-known JA model. It is one drawback of the model that it is difficult to specify core parameters from measured hysteresis loops.

* Corresponding author.
which makes it less attractive from a simulation point of view. Therefore, Jiles et al. proposed a method for the numerical determination of hysteresis parameters from measured curves [4]. A commercial program called Magpack is available for calculating the model parameters of this model. In Ref. [17] a piecewise linear ladder circuit has been presented which exhibits hysteresis. The hysteresis curve of this model is piecewise smooth so that convergence problems might occur at the transition points. The first-derivative w.r.t. $M$ is not continuous. Hence, the induced voltage at the ports of an inductor which is proportional to $dM/dH$ is not steady. In the papers of Hodgdon et al. [10–12] an empirical model has been proposed. Frequency dependence has been included too by additional terms in $B$. The more recent variants of the model [10,12] are based on a differential equation in $dH/dB$. Hence, cause and effect is interchanged. It is therefore very difficult to compare Hodgdon’s model with the model illustrated here which models $dM/dH$ by a system of differential equations.

In this paper a really simple though accurate model is proposed. Saturation magnetization and coercivity are direct parameters of the model. The other ones are obtained subsequently from the remanence magnetization and the slope of the initial curve at the origin and the slope at remanence. The model exhibits all main features of hysteresis such as initial magnetization, saturation, coercivity, remanence and frequency-dependent losses. It consists merely of three differential equations with six parameters. Two nonlinear ordinary differential equations (ODE’s) model the static behavior and one linear ODE the frequency-dependent losses. This ODE can be omitted. Then the model only exhibits the static behavior of hysteresis. Four variants of the model are proposed which can be selected depending on the slope of the outer hysteresis loop. It is based on experiences obtained from the JA model.

This paper is organized as follows. In Section 2 the novel empirical model is presented. It is highly accurate and parameter determination is a simple task. Section 3 deals with the parameter determination for this empirical model and in Section 4 simulation results are presented.

### 2. An empirical hysteresis model

The reasons why it is not easy to determine core parameters from measured devices for the JA model stem from the fact that the rate of change of $M$ is proportional to the distance of the actual magnetization to an ideal equilibrium state which is referred to as the anhysteretic magnetization $M_{an}$. $M_{an}(H)$ cannot be measured directly. Furthermore, a mean field $H_{m}$ is introduced which describes the coupling of domains. However, $H_{m}$ cannot be measured either. By contrast the modeling of hysteresis via a parameter dependent nonlinear ordinary differential equation in $H$ for the magnetization $M$ is a flexible and hence very promising approach.

Therefore, the empirical model illustrated below models the hysteresis by a nonlinear differential equation for the rate of change of $M$. This function depends directly on $H$. Hence, the parameters of this function can be determined directly from measured data.

*Note that in what follows the magnetization $M$ is normalized to the saturation magnetization $M_s$. The derivation of the model starts with the modeling of the full outer loop. $dM/dH$ for the outer loop takes the following structure:

$$
\frac{dM}{dH} = K^{-1} g(H, H_c),
$$

where $H_c$ is the coercivity and $g(\cdot, \cdot)$ is a function with the following properties:

$$
\lim_{H \rightarrow \pm \infty} g(H, 0) = 0, \quad (2)
$$

$$
g(-H, 0) = g(H, 0), \quad (3)
$$

$$
\int_{-\infty}^{\infty} K^{-1} g(H, 0) \, dH = 2. \quad (4)
$$

$g(\cdot, \cdot)$ models the outer hysteresis loop. Here we propose the following empirical functions for $g(\cdot, \cdot)$:

$$
g(H, H_c) = \frac{1}{1 + c[H - H_c \, \text{sign}(dH/dt)]^2}, \quad \alpha > 1, \quad (5)
$$

$$
g(H, H_c) = \exp(-c[H - H_c \, \text{sign}(dH/dt)]^2), \quad (6)
$$
with parameters $c$ and $H_c$. The last function is a shifted Gaussian function.

We consider here only the cases $x = 2, 3, 4$ and the Gaussian function, i.e.

$$ g(H, H_c) = \frac{1}{1 + c (H - H_c \text{sign}(dH/dt))^3}, \quad (7) $$

$$ g(H, H_c) = \frac{1}{1 + c |H - H_c \text{sign}(dH/dt)|^3}, \quad (8) $$

$$ g(H, H_c) = \frac{1}{1 + c (H - H_c \text{sign}(dH/dt))^2}, \quad (9) $$

$$ g(H, H_c) = \exp(-c(H - H_c \text{sign}(dH/dt))^2). \quad (10) $$

The four alternatives given in Eqs. (7)–(10) fulfill the properties (2) and (3). Note that $g(\cdot, \cdot)$ is symmetric around $H_c$.

The determination of $c$ from measured curves is a simple task which is illustrated in the next section. Which function is best depends on the hysteresis under consideration. Solving Eq. (4) for the different formulas (7)–(10), $K$ takes the values [16]:

$$ K = \begin{cases} \frac{\pi}{2\sqrt{c}}, \\ \frac{1}{\sqrt{3\sqrt{3/c}}} \left(\frac{\pi}{2} + \arctan(1/\sqrt{3})\right), \quad = \frac{2\pi}{3\sqrt{3\sqrt{c}}}, \\ \frac{\pi}{2\sqrt{2\sqrt{c}}}, \\ \frac{\pi}{4c} \end{cases} \quad (11) $$

The rate of change $|dM/dH|$ is smaller when minor loops occur. Hence Eq. (1) has to be modified for the modeling of minor loops

$$ \frac{dM}{dH} = K^{-1} f(M, M_0, H)g(H, H_c). \quad (12) $$

$f(\cdot, \cdot, \cdot)$ depends on $dH/dt$. If $\text{sign}(dH/dt) = 1$ the function $f$ takes the value unity at the lower outer hysteresis curve and the value zero for $M = 1$ ($M$ has been supposed to be normalized to $M_0$). For being symmetric $f$ takes the value unity at the upper hysteresis and zero for $M = -1$ if $\text{sign}(dH/dt) = -1$. We consider here only the case $\text{sign}(dH/dt) = 1$. The derivation of $f(\cdot, \cdot, \cdot)$ starts with the introduction of a second differential equation which causes saturation,

$$ \frac{dM/dH}{H} = K^{-1} g(H, 0). \quad (13) $$

$M_0(H)$ takes the same slope as the major loops. In fact, the major loops are only shifted functions of $M$. $M_0$ divides the outer hysteresis loop into a lower and an upper branch. Note that $M_0$ can also be used alternatively as the anhysteresis function of the JA model.

The function $f$ used here looks as follows (where $\text{sign}(dH/dt) = 1$):

$$ f(M, M_0, H) = 1 - f_2(M, M_0, H) $$

$$ \cdot (\beta + (1 - \beta) f_3(M, M_0)) \quad (14) $$

with the property that $f_3(M_0, M_0, H) = 1$ and $f_3(M_0, M_0) = 0$. Hence, at the origin, the rate of change is $dM/dH|_{M = 0, H = 0} = (1 - \beta) K^{-1} g(H, 0)$. Therefore, $\beta$ is directly obtained from measured curves where the parameter $c$ of Eq. (7) has been evaluated a priori. $f_3(\cdot, \cdot, \cdot)$ models the minor loops at the upper and $f_3(\cdot, \cdot, \cdot)$ at the lower branch. Therefore $f_3 = 1$ for $M > M_0$ and $f_3 = 0$ otherwise. The condition that $f$ takes the value zero for $M = 1$ can be met by

$$ f_3(M_0, M) = \begin{cases} (M - M_0)/(1 - M_0) \quad (M - M_0) > 0, \\ 0 \quad \text{elsewhere} \end{cases} \quad (15) $$

1 For the more general function (5) with the additional parameter $\alpha$ the constant $K$ has to be calculated numerically.
and that \( f \) takes the value unity at the lower major branch is sufficiently met by
\[
f_2(M, M_0, H) = \begin{cases} 
1 & x < 0, \\
1 - x/d & 0 < x < d, \\
0 & x \geq d,
\end{cases}
\] (16)

with a suitably chosen parameter \( d \), where \( x = (M_0 - M)/g(H, 0) \) in the equation above. The additional parameters \( \beta \) and \( d \) are calculated directly from the rate of change at the origin and the negative remanence. This is illustrated in the next section.

Frequency dependence is included by replacing \( H \) by \( H_i \) in the equations above, where \( H_i \) is calculated by an ordinary differential equation
\[
\frac{dH_i}{dt} = \frac{H - H_i}{K_t}.
\] (17)

\( K_t \) is a material dependent parameter. The idea behind Eq. (17) is to perform a basis transformation depending on the rate of change of the magnetic field strength \( H \). At low frequencies the dynamic model converges to the static one.

The entire model consists of the Eqs. (7)–(17) with the six parameters \( M_0, H_c, c, \beta, d, K_t \).

### 3. Parameter determination

The hysteresis model described here is compared solely of five parameters. The saturation magnetization \( M_s \) and the coercivity \( H_c \) are determined directly from measured devices. The parameter \( c \) of the alternative functions \( g(\cdot, \cdot) \) (7) is calculated by
\[
M(H_0 + H_c, \text{sign}(\dot{H}) = 1) - M(-H_0 + H_c, \text{sign}(\dot{H}) = 1).
\] (18)

\( M(H_0 + H_c) \) and \( M(-H_0 + H_c) \) are obtained directly from measured hysteresis loops. The solution of the integral on the left-hand side can be found in various textbooks, e.g. Ref. [16]. Note that \( K \) is calculated for the four variants of the model by Eq. (11). The choice of \( H_0 \) is free and left to the user. One useful choice is \( H_0 = H_c \). Then the calculated hysteresis curve meets the measured magnetization at remanence.\(^2\) For the more general approximation (5) with the additional parameter \( x \) the integral at the left-hand side of Eq. (18) must be calculated numerically.

The remaining parameters \( \beta \) and \( d \) are computed with the aid of the rate of change at the origin and the negative remanence:
\[
1 - \beta = \frac{dM/dH|_{M=0,H=0}}{K^{-1}g(H_c, 0)},
\] (19)
\[
\beta = \frac{1}{d} \frac{(dM/dH|_{M=-M_c,H=0} - dM/dH|_{M=0,H=0})}{K^{-1}g(H_c, 0)},
\] (20)

where \( M_t \) is the magnetization at remanence. Eq. (20) follows directly from Eqs. (12), (14)–(16) and (19) for \( M = -M_c \). Parameter determination is therefore a simple task because parameter \( c \) is calculated directly from the remanence magnetization using Eq. (18) by setting \( H_0 = H_c \), \( \beta \) from the slope at origin (19) and \( d \) from the slope at origin and remanence (20). An easy determination of the parameter \( K_t \) which models frequency dependence is still unexplored.

### 4. Results

The hysteresis model described here is compared with the results obtained with the JA model [2]. In Ref. [4] Jiles et al. derived a numerical determination process of the hysteresis parameters for their model of differential-algebraic equations. The data obtained there has been taken as the basis of our parameter evaluation. Table 1 shows the parameters for different magnetic materials.

Fig. 1 compares the hysteresis for the Mn–Zn ferrite using the JA-model and the empirical model proposed here using the variant (7). The coercivity and remanence is in good agreement, whereas the

\[^2\] The choice of \( H_0 \) effects the value \( c \) and therefore the slope of the loop.
Table 1
Parameter of the JA model for different magnetic devices according to Ref. [4]

<table>
<thead>
<tr>
<th>Material</th>
<th>Ferrite</th>
<th>Fe 1.0 wt% C</th>
<th>Fe 0.6 wt% C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_s$ (A/m)</td>
<td></td>
<td>$1.5 \times 10^6$</td>
<td>$1.6 \times 10^6$</td>
</tr>
<tr>
<td>$a$</td>
<td>1800</td>
<td>972</td>
<td>27</td>
</tr>
<tr>
<td>$k$</td>
<td>1800</td>
<td>672</td>
<td>30</td>
</tr>
<tr>
<td>$\chi$</td>
<td>$1.4 \times 10^{-3}$</td>
<td>$1.4 \times 10^{-3}$</td>
<td>$5^{-5}$</td>
</tr>
<tr>
<td>$c$</td>
<td>0.14</td>
<td>0.14</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Fig. 1. Comparison of the simulated curves for a Mn-Zn ferrite using the model [2] (dashed) and the one of this paper given by Eq. (7) (solid). The parameters are shown in Tables 1 and 2, respectively.

transition to saturation is sharper for this parameter set. Fig. 2 compares both models for a Fe 0.6 wt% carbon steel using also the variant (7). Again coercivity and remanence are in good agreement and the transition to saturation is sharper. In fact, for a field strength of $H = 10$ kA/m the magnetization of the JA-model is roughly 90% and of the model illustrated here 98% of the saturation. Figs. 3 and 4 compare both models for a Fe 1.0 wt% carbon steel with variant (7) and (8), respectively. Especially the variant (7) leads to a very good agreement of both models. Table 2 summarizes the fitted parameters for the empirical model. The empirical model leads often to a sharper transition to saturation compared with JA. This transition can be smoothed using Eq. (5) with an $1 < \chi < 2$.

Fig. 2. Comparison of the simulated curves for a Fe 0.6 wt% C carbon steel using the JA model (dashed) and the variant given by Eq. (7) (solid). The parameters are shown in Tables 1 and 2, respectively.

Fig. 3. Comparison between the JA model (dashed) and the variant given by Eq. (7) (solid) for a Fe 1.0 wt% C carbon steel. The parameters are shown in Tables 1 and 2, respectively.

Fig. 5 illustrates a theoretical hysteresis loop using $g(H, 0) = 1/(1 + cH^4)$ where $\beta = 0.7$, $d = 0.3$, $H_{c} = 60$ A/m, $c = 1.0 \times 10^{-7}$, $K_{r} \rightarrow 0$. The transition to saturation starts abruptly and $M_s/M_r \approx 0.9$. Hence, this variant of the model is preferred for hard magnetic materials. The Figs. 6–8 show simulated hysteresis curves using the Gaussian function. The coercivity changes from $H_{c} = 20$ A/m for a typical soft magnetic material to $H_{c} = 1.0 \times 10^5$ A/m
Fig. 4. Comparison between the JA model (dashed) and the variant given by Eq. (8) (solid) for a Fe 1.0 wt% C carbon steel. The parameters are shown in Tables 1 and 2, respectively.

Table 2
Parameter of the empirical model for the same materials given in Table 1. The Fe 1.0 wt% C has been modeled by two variants given in the text

<table>
<thead>
<tr>
<th>Material</th>
<th>Variant</th>
<th>( M )</th>
<th>( H_c )</th>
<th>( K )</th>
<th>( \beta )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe 1.0 wt% C</td>
<td>Eq. (7)</td>
<td>( 1.5 \times 10^6 )</td>
<td>( 1.5 \times 10^6 )</td>
<td>4057</td>
<td>0.84</td>
<td>0.26</td>
</tr>
<tr>
<td>Fe 1.0 wt% C</td>
<td>Eq. (8)</td>
<td>( 1.6 \times 10^6 )</td>
<td>( 620 )</td>
<td>5813</td>
<td>0.83</td>
<td>0.3</td>
</tr>
<tr>
<td>Fe 0.6 wt% C</td>
<td>Eq. (7)</td>
<td>( 0.4 \times 10^6 )</td>
<td>( 2.8 \times 10^{-6} )</td>
<td>938</td>
<td>0.96</td>
<td>0.26</td>
</tr>
<tr>
<td>Mn-Zn ferrite</td>
<td>Eq. (7)</td>
<td>( 0.7 )</td>
<td>( 3.0 \times 10^{-4} )</td>
<td>91</td>
<td>0.7</td>
<td>0.1</td>
</tr>
</tbody>
</table>

for a hard magnetic one. This demonstrates that the model copes with both soft and hard magnetic materials. The example depicted in Fig. 9 compares the static behavior \( (K_f \to 0) \) (continuous line) with the frequency-dependent model \( (K_f = 5.0 \times 10^{-7}) \) (dashed line). The loop tips at high frequencies are smoother and the coercivity increases which leads to an increase of the energy losses too.

5. Conclusions and future work

In this paper a hysteresis model has been presented which exhibits all the main features of hysteresis such as saturation, coercivity, remanence, frequency dependence and hysteresis losses. The differential equations which model the hysteresis are fully empirical. The goal of this development was to find a set of differential equations where the model parameters can be obtained directly from measured curves, especially the saturation magnetization, coercivity and remanence as well as the
initial slope. Furthermore, besides turning points the hysteresis loops are sufficiently smooth which improves the convergence behavior in a practical application. This capability has been demonstrated by comparing simulated hysteresis loops of this model with a classical model well-known in literature. The more general function (5) for the right-hand side of the differential Eq. (1) is more flexible due to the additional parameter $a$ and copes therefore better with measured hysteresis loops. The authors are currently exploring a simple numerical method for the determination of the parameters of this equation. The same is true for the parameter $K_f$ of the frequency dependency (17).

References
